Soft Computing Applications (IT 60108)
Spring, 2015-2016

Practice Sheet – I

(Fuzzy Logic)

1. Suppose A is a fuzzy set defined over a universe of discourse X. If $Core(A)$ denotes the core of the fuzzy set A, then $Core(A)$ is a crisp set. What about the $Support(A)$?

2. For a singleton fuzzy set A, how many crossover point(s) is(are) possible?

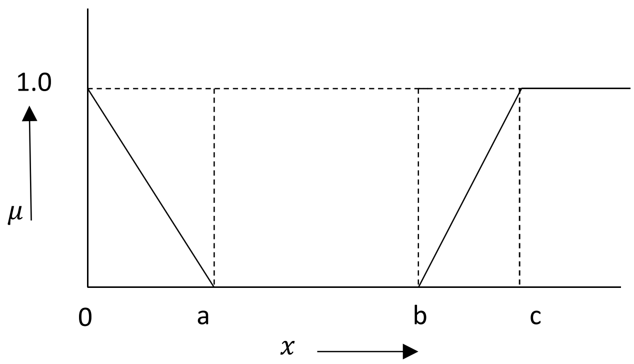
3. A crisp set A defined over $X = \{1,2,3,5,7\}$ is $A = \{1,3,7\}$. What would be it's equivalent fuzzy set?

4. A fuzzy set is given by $B = \{(1,0.1),(2,0.2),(3,0.3),(4,0.9),(0,0.0)\}$. What is the crisp set that can be concluded from it?

5. A membership function $\mu(x)$ is defined as $\mu(x) = \frac{x}{1-x}$ $\forall\, x \in R$, R is the set of all real numbers. Check whether $\mu(x)$ can qualify as a member function.

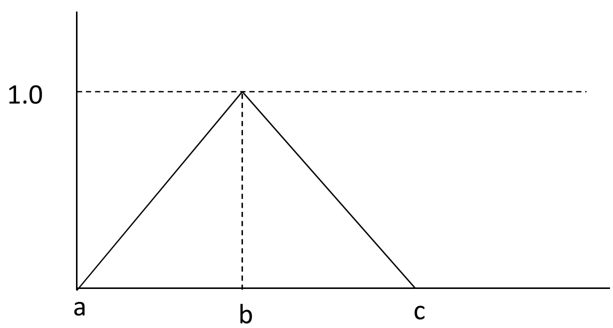
6. $\forall\, x \in R$, the membership function is defined as $\mu(x) = \frac{x}{1+x^2}$. Determine whether $\mu(x)$ is open left, open right or closed.

7. A membership function is graphically shown as follows:



Let this membership function be denoted as Cup. Define $Cup(x: a, b, c)$

8.



$$\mu(x) = \begin{cases} 0, x < a \\ \frac{x-a}{b-a}, a \leq x < b \\ \frac{c-x}{c-b}, b \leq x \leq c \\ 0, x > c \end{cases}$$

Redefine $\mu(x)$ when $\mu(x) = \alpha$ and $0 < \alpha < 1$.

9. Composition of two fuzzy relations R and S is

$R°S = \{\{(x,z)|(x,y) \in R \; and \; (y,z) \in S\}\}$. Let $T = R°S$, then T can be defined with two rules:

Rule 1: $T(x,z) = max\{min\{R(x,b), S(y,z)\}\forall \; y \in Y\}$ (Max-min composition)

Rule 2: $T(x,y) = max\{\{R(x,y)° \; S(y,z)\}\forall \; y \in Y\}$ (Max-product also called max-dot composition)

(a) Interpret the physical implication of the above mentioned two relations.

(b) Also check if $R°S = S°R$ according to each rule.

10. Three fuzzy sets are given as follows:

$$A = \left\{\frac{0.5}{Winter}, \frac{0.33}{Spring}, \frac{0.52}{Summer}, \frac{0.25}{Fall}\right\}$$

$$B = \left\{\frac{0.10}{Winter}, \frac{0.55}{Spring}, \frac{0.90}{Summer}, \frac{0.20}{Fall}\right\}$$

$$C = \left\{\frac{0.22}{High}, \frac{0.55}{Medium}, \frac{0.44}{Low}\right\}$$

Derive the following relations:

(a) If x is A or y is B then z is C.

(b) If x is A and y is $\sim B$ then z is C.

11. Zadeh's max-min rule is defined as:

$R = \bar{A} \cup (A \cap B) = \int_{X \times Y} (1 - \mu_A(x)) \cup (\mu_A(x) \cap \; \mu_B(y))|(x,y)$ so that

$R'_{MM} = (A \times B) \cup (\bar{A} \; \times Y)$ is equivalent to R_{MM}.

12. Verify the following (according to Zadeh's max-min rule)

(a) If x is A then y is $B \equiv (A \times B) \cup (\bar{A} \times Y)$

(b) If x is A then y is B else z is $C \equiv (A \times B) \cup (\bar{A} \times C)$
(Verify with the fuzzy sets in Question 10)

13. What is the minimum and maximum number of crossover points possible for a fuzzy set?

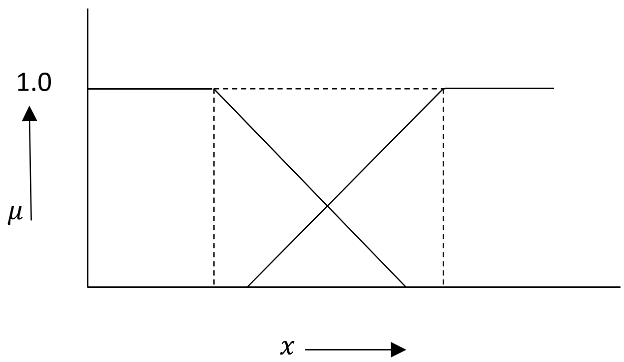
14. Decide whether the following fuzzy sets are closed or open?

(a) $\mu_A(x) = \frac{x}{1+x}$

(b) $\mu_A(x) = 2^{-x}$

(c) $\mu_A(x) = \frac{x}{1+x^2}$

(d) $\mu_{A\cap B}$ and $\mu_{A\cup B}$ for μ_A and μ_B with the following MFs:



15. For grading system, draw membership functions for the following:

(a) Crisp grading.

(b) Fuzzy grading with trapezoidal MF.

(c) Fuzzy grading with gaussian MF.

(You can make a reasonable assumption, if any.)

16. What is the physical representation of the fuzzy set $A \leftarrow B$, where A and B are two fuzzy sets defined over a universe of discourse X.

17. Give an example where you must follow Soft computing methods instead of Hard computing. Justify your answer.

18. What are the three characteristics that Hard computing must obey?

19. Why Soft computing is preferable than Hard computing to solve some problems? Give examples for each which you should consider for solving:

(a) Using Soft computing only.

(b) Using Hard computing only.

(c) Using both, Soft and Hard computing.

20. For the following fuzzy set A defined over a universe of discourse, draw the graph.

$X = \{10, 15, 20, 25, 30, 35, 40, 45, 50\}$
$A = \{(15,0.5), (20,0.4), (25,0.3), (30,0.6), (35,0.8)\}$

21. Given two fuzzy sets A and B, defined over the universe of discourses X and Y respectively. Draw the graphs for the following:

(a) $A \times B$

(b) $R: A \rightarrow B$

Given,

$A = \{(20,0.2), (25,0.4), (30,0.6), (35,0.6), (40,07), (45,0.8), (50,0.8)\}$

$B = \{(1,0.8), (2,0.8), (3,0.6), (4,0.4)\}$

$X = \{10, 15, 20, 25, 30, 35, 40, 45, 50\}$

$Y = \{1, 2, 3, 4\}$

(c) If x is A then y is B. What rule says if $x = 40$?

22. Prove (or disprove) the validity of the following:

(a) $P \rightarrow Q \equiv \sim P \cup Q$

(b) If $P \rightarrow Q$ then $Q \rightarrow P$ (Contrapositive)

(c) If $P \rightarrow Q$ then $\sim Q \rightarrow \sim P$ (Contranegative)

(d) If P and $P \rightarrow Q$ then

P	Q	$P \rightarrow Q$
F	F	T
F	T	T

$P \rightarrow Q \equiv \sim P \cup Q$

$P, P \rightarrow Q \equiv Q$

T	F	F
T	T	T

Modus Ponens (MP): $P, P \rightarrow Q \Rightarrow Q$

Modus Tollens (MT): $P \rightarrow Q, \sim Q \Rightarrow \sim P$

Hypothetical Syllogism (HS): $P \rightarrow Q, Q \rightarrow R \equiv P \rightarrow R$

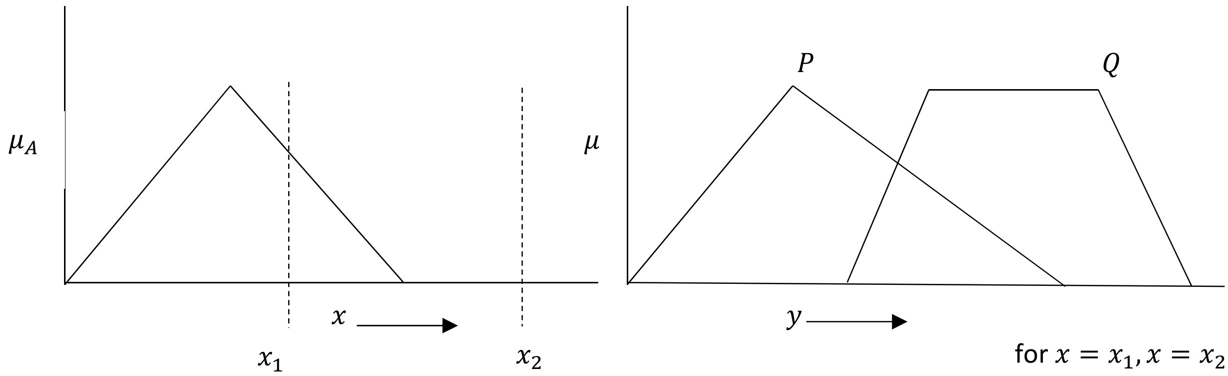
Constructive Dilemma (CD): $P \cup Q, P \rightarrow R, Q \rightarrow S \equiv R \cup S$

23. What will be the output fuzzy set in the following cases:

(a) If x is A then y is B

(b) $\left\{\begin{array}{l}\text{If } x \text{ is } A \text{ then } y \text{ is } P \\ \text{If } x \text{ is } B \text{ then } y \text{ is } Q\end{array}\right.$

(c)



for $x = x_1, x = x_2$

24. **State true or false:**

(a) If $\lambda \leq \propto$ then $A_\alpha \subseteq A_\lambda$ where $\alpha, \lambda \in [0,1]$ and A_α, A_λ denotes $\alpha - and\ \lambda -$ cuts of the fuzzy set A.

(b) If A_α denotes the $\alpha -$ cut of a fuzzy set, then A_0 is the universe of discourse of fuzzy set A.

(c) If A_α denotes the $\alpha -$ cut of a fuzzy set, then $A_1 = Support(A)$ or $A_1 = Core(A)$.

(d) For any two fuzzy sets A and B defined over a universe of discourse X,

$A \oplus B = (\bar{A} \cap B) \cup (A \cap \bar{B})$

$$A - B = A \cap \bar{B}$$

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x)$$

What is $\mu_{A-B}(x)$?

25. What is the output of the fuzzy element $p: x\ is\ F$

(a) a fuzzy set.

(b) a fuzzy rule.

(c) $\mu_F(x)$, the membership value of x.

(d) p is a rule matrix.

26. Given two fuzzy sets A and B, defined over the universe of discourses X and Y respectively, and a rule $R : If\ x\ is\ A\ then\ y\ is\ B$. How would you calculate the following:

(a) $x\ is\ A\ given\ that\ y\ is\ B$

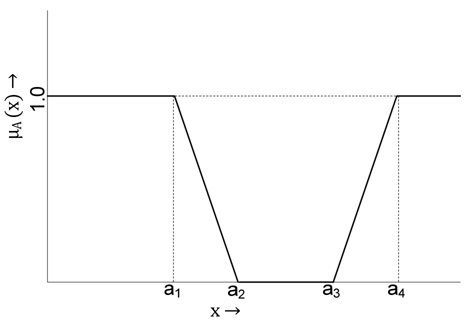
(b) $y\ is\ B'\ given\ that\ x\ is\ A$

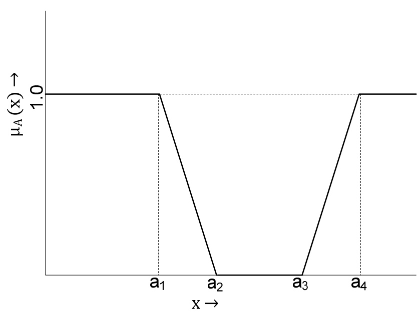
(c) $x\ is\ A'\ given\ that\ y\ is\ B'$

(d) $x\ is\ A'\ given\ that\ y\ is\ B$

27. How you can obtain the crisp value of a fuzzy set A whose membership function is defined as $\mu_A(x) = \frac{1}{1+x^2}$

28. Two fuzzy sets A and B are defined over a universe of discourse X with membership functions $\mu_A(x)$ and $\mu_B(x)$, respectively as shown below





Assume that $a_1 = b_2$ and $a_2 = b_3$

Draw the plot of membership functions of $A \cup B$, $A \cap B$ and $\bar{B}$.

29. Consider three sets as stated below (In the context of courses offered among students)

$S = \{s_1, s_2, s_3, s_4\}$ is a set of students

$C = \{c_1, c_2, c_3\}$ is a set of courses

$P = \{p_1, p_2, p_3, p_4\}$ denotes a set of level of popularity

Two relations are given below:

$R_1 =$

	c_1	c_2	c_3
s_1	0.1	0.2	0.3
s_2	0.2	0.3	0.1
s_3	0.2	0.3	0.1
s_4	0.3	0.1	0.2

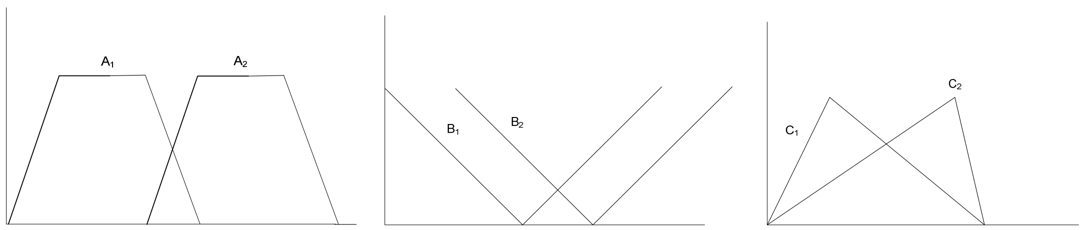
$R_2 =$

	p_1	p_2	p_3	p_4
c_1	0.4	0.3	0.2	0.5
c_2	0.1	0.3	0.5	0.7
c_3	0.2	0.4	0.6	0.8

(a) Find $R_1 \text{ o } R_2$.

(b) What are the physical implementation of R_1, R_2 and $R_1 \text{ o } R_2$?

30. The membership distribution functions for the fuzzy sets A_1 A_2, B_1 B_2, C_1 C_2 are shown in the graphs below:



The above fuzzy system is subjected to the satisfaction of the following rules for two fuzzy inputs $x = \alpha$ and $y = \beta$.

R_1: IF x is A_1 AND y is B_1 THEN z is C_1

R_2: IF x is A_2 AND y is B_2 THEN z is C_2

(a) Calculate the fuzzy output.

(b) Also, show how the fuzzy output can be defuzzified into its crisp value using center of sum (COS) method.

31. The ***support*** of Fuzzy set A is the set of all points *x* in *X* (is the universe of discourse) such that

(a) $\mu_A(x) > 0$

(b) $\mu_A(x) = 1$

(c) $\mu_A(x) = 0.5$

(d) $\mu_A(x) \neq 1$

32. Takugi-Sugeno approach to FLC design is computationally more expensive compared to Mamdani approach because;

(a) Mamdani approach considers a less number of rules in fuzzy rule base.

(b) Searching a rule in Mamdani approach is simple and hence less time consuming.

(c) Takagi-Sugeno approach consider a large number of rules in fuzzy rule base.

(d) Computation of each rule in Takagi-Sugeno approach is more time consuming.

33. A membership function μ_x is defined as $\mu_x = \lceil x \rceil$. The membership function is with

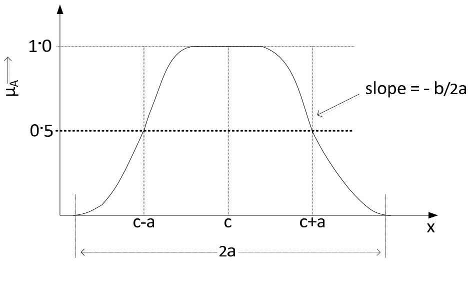
(a) Discrete values over a discrete domain of universe of discourse.

(b) Discrete values of a continuous domain of universe of discourse.

(c) Continuous values of a discrete domain of universe of discourse.

(d) Continuous values of a continuous domain of universe of discourse.

34. Figure below shows the membership function $\mu_A(x)$ of a fuzzy set *A* defined on a universe of discourse *X*.



(a) Mark the core, crossover point(s) and support of the fuzzy set *A.*

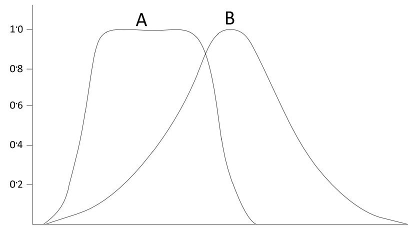
(b) With parameters given in Fig above, formulate $\mu_A(x)$ as a *generalized bell MF*.

35. The membership functions of two fuzzy sets *A* and *B* is shown in figure below. Draw graphically the fuzzy sets for the following.

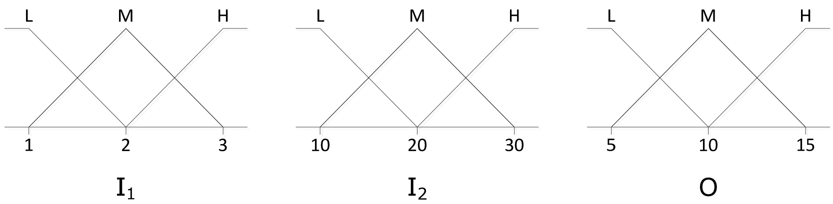
(a) Intersection of *A* and *B*

(b) Union of *A* and *B*

(c) Compliment of *A* and *B*



36. There are two inputs I_1 and I_2 and an output *O* of a process. It is required to develop a fuzzy logic controller (FLC) based on the *Mamdani approach*. The inputs and output are expressed using three linguistic terms namely *L* (low), *M* (medium) and *H* (high). The membership function distributions of the above inputs and output are shown in Figure. The rule-base of the fuzzy logic controller is shown in Table below:



Table

		I_2		
		L	M	H
	L	L	L	M
I_1	M	L	M	H
	H	M	H	H

Suppose, at any instant, inputs to the fuzzy logic controller are $I_1 = 1.5$ and $I_2 = 25$.

(a) Obtain the *fuzzified* values of the input.

(b) Compute rule strengths of the rules corresponding to the given inputs.

(c) Decide the fuzzy output for the given inputs.

(d) *Defuzzyfying* the output using *Center of Sum* (COS) method.